

Exponents, Radicals, and Scientific Notation

General Exponent Rules:

$$x^m \times x^n = x^{m+n} \quad \text{Example 1: } x^5 \times x^2 = x^{5+2} = x^7$$

$$(x^m)^n = x^{mn} \quad \text{Example 2: } (x^5)^2 = x^{5 \times 2} = x^{10}$$

$$(x^m y^n)^p = x^{mp} y^{np} \quad \text{Example 3: } (3x)^2 = 3^2 x^2 = 9x^2$$

$$x^0 = 1 \text{ if } x \neq 0 \quad \text{Example 4: } 56^0 = 1$$

$$x^{-n} = \frac{1}{x^n} \quad \text{Example 5: } 2a^{-4} = \frac{2}{a^4}$$

$$\frac{1}{x^{-n}} = x^n \quad \text{Example 6: } \frac{x}{y^{-5}} = xy^5$$

$$\frac{x^m}{x^n} = x^{m-n} \quad \text{Example 7: } \frac{x^7}{x^5} = x^{7-5} = x^2$$

Scientific Notation:

Scientific Notation is a way to write very large and very small numbers more easily. For example Avogadro's number, often used in chemistry and physics, is 602,200,000,000,000,000,000 but in scientific notation the number is 6.022×10^{23} . To convert a number into scientific notation, you want to move the decimal point so that only one non-zero digit is in front of the decimal point. Then count the number of places you moved the decimal point. That number will be the superscript above the $\times 10$. If you moved the decimal to the left, the superscript will be positive. If you moved the decimal to the right, the superscript will be negative. To convert from scientific notation to decimal notation, move the decimal point to the right by the value of the superscript if the superscript is positive. You will move the decimal point to the left if it is negative.

Example 8: Write 45,667,323.003 in scientific notation.
 4.5667323003×10^7

Example 9: Write 0.00000000004503 in scientific notation.
 4.503×10^{-11}

Example 10: Write 3.405×10^{-4} in decimal notation.
0.0003405

Simplifying Radicals:

First, work with the numbers and variables separately. For the numbers, factor them into prime numbers, and then write them under the radical so that the same numbers are next to each other. You will then group the like numbers together, to determine how many like numbers go in each group you will look for the root (this is the number inside the check of the radical). For each group, you can take one of the numbers in the group out of the radical. What does not fit in a group stays inside the radical. With the variables, divide their power (exponent) by the root. The number of times the root goes into the power is the power of the variable outside of the radical. The remainder is the power of the variable that stays inside.

Example 11: $\sqrt{75 m^5}$

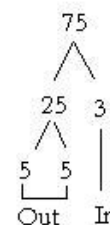
•Note: the group size for this radical is two because we are using a square root.

Step 1: find the prime factors of 75 and rewrite them in the radical.

$$\sqrt{5 \times 5 \times 3 \times m^5}$$

Step 2: Pull out the group of two 5s.

$$5\sqrt{3 \times m^5}$$



Step 3: Divide the power of m (5) by the value of the root (2). Remember, the whole number is the power of the variable that comes out of the radical and the remainder is the power of the variable that remains inside the radical.

$$5 \div 2 = 2 \text{ R } 1$$

$$5m^2\sqrt{3m^1}$$

Example 12: $\sqrt[3]{162x^2y^{16}z^3}$

•Note: the group size for this radical is three because we are using a cubed root.

Step 1: Factor 162 into prime factors and write them in order under the radical.

$$\sqrt[3]{2 \times 3 \times 3 \times 3 \times 3 \times 3 \times x^2y^{16}z^3}$$

Step 2: Pull out the group of three 3s.

$$3\sqrt[3]{2 \times 3 \times x^2y^{16}z^3}$$

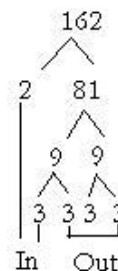
Step 3: Divide the exponents by the root.

$$x: 2 \div 3 = 0 \text{ R } 2$$

$$y: 16 \div 3 = 5 \text{ R } 1$$

$$z: 3 \div 3 = 1 \text{ R } 0$$

$$3y^5z^1\sqrt[3]{6x^2y^1}$$



Adding or Subtracting Radicals:

First, simplify each radical. Then add or subtract like terms. “Like terms” are those terms that have the same radicals and variables. This means that only radicals with the same root, the same variables and numbers on the inside of the radical, and the same variables on the outside of the radical may be added or subtracted together.

Example 13: •Note that you cannot add square roots and cube roots.

$$\begin{array}{l}
 y\sqrt{75x} + 3\sqrt{3xy^2} - y^3\sqrt{81x} \\
 5y\sqrt{3x} + 3y\sqrt{3x} - 3y^3\sqrt{3x} \\
 (5y + 3y)\sqrt{3x} - 3y^3\sqrt{3x} \\
 8y\sqrt{3x} - 3y^3\sqrt{3x}
 \end{array}
 \qquad
 \begin{array}{c}
 75 \\
 \wedge \\
 25 \times 3 \\
 \wedge \quad \backslash \\
 \underline{5 \times 5} \times 3
 \end{array}
 \qquad
 \begin{array}{c}
 81 \\
 / \quad \backslash \\
 9 \times 9 \\
 \wedge \quad \wedge \\
 \underline{3 \times 3} \times \underline{3 \times 3}
 \end{array}$$

Multiplying Radicals:

First, check to see if the two radicals have the same root. If so, multiply the radicals together. Multiply outside terms by outside terms and inside terms times by inside terms. Then simplify. If the roots are not the same you cannot multiple them together.

Example 14: $(2\sqrt{8})(3\sqrt{2})$
 $6\sqrt{16}$
 6×4
 24

Dividing Radicals:

Radicals are not allowed in the denominator of a fraction. Therefore we must remove them. To do this we use a process called “Rationalizing the Denominator.” In other words, we will multiply both the numerator and the denominator by a radical with the same index in order to remove the radical from the denominator.

Example 15: $\frac{3x}{\sqrt{5}}$
 $\frac{3x}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{3x\sqrt{5}}{5}$

Example 16: $\frac{2}{\sqrt[3]{4x}}$

$$\frac{2}{\sqrt[3]{4x}} \times \frac{\sqrt[3]{16x^2}}{\sqrt[3]{16x^2}} = \frac{2\sqrt[3]{2^4x^2}}{\sqrt[3]{2^6x^3}} = \frac{4\sqrt[3]{2x^2}}{4x} = \frac{\sqrt[3]{2x^2}}{x}$$

To simplify (or divide) a fraction with a square root radical and another term in the denominator, you must multiply both the numerator and the denominator by the conjugate of the denominator. Then simplify.

Example 17: $\frac{2}{\sqrt{5}+1}$

$$\frac{2}{\sqrt{5}+1} \times \frac{\sqrt{5}-1}{\sqrt{5}-1} = \frac{2(\sqrt{5}-1)}{5-1} = \frac{2(\sqrt{5}-1)}{4} = \frac{2(\sqrt{5}-1)}{4 \cdot 2} = \frac{\sqrt{5}-1}{2}$$

Sample Problems:

Simplify each of the following completely.

1. $(-1)^6$

2. $(-5)^0$

3. $\left(\frac{1}{3}\right)^4$

4. -2^4

5. $-2(-5)^2$

6. $(10^2)^3$

7. $2^2 2^3$

8. $a^5 a^4 a^0$

9. $(4a^2)^{\frac{3}{2}}$

10. $\left(\frac{x^2}{y^3}\right)^2$

11. $\sqrt{\frac{4}{81}}$

12. $\sqrt{a^2}$

13. $\sqrt{64x^4w^6}$

14. $\sqrt[3]{y^3}$

15. $\sqrt[3]{-0.008}$

16. 5^{-2}

17. $16^{\frac{3}{2}}$

18. $(-8)^{\frac{1}{3}} + (-8)^{\frac{2}{3}}$

19. $9^{\frac{-1}{2}}$

20. $\left(\frac{1}{5}\right)^{-1}$

21. $x^0 x^{\frac{5}{3}} x^{\frac{1}{6}}$, $x \geq 0$

22. $\frac{2a^{-1}}{3b^{-2}}$

23. $\frac{2^{-2}-3^{-2}}{2^{-2}3^{-2}}$

24. $\frac{c^{-1}-d^{-1}}{c^{-2}-d^{-2}}$, $c \neq \pm d$, $c \neq 0$, $d \neq 0$

25. $(a^{-2}y^{-3})^{-1}$, $a \neq 0$, $y \neq 0$

26. $(4y)^{-2} \cdot \frac{y^4}{y^2}$, $y > 0$

27. $(8a^{-3}y^6)^{\frac{2}{3}}$

28. $\sqrt{18x^3y^4}$, $x \geq 0$

29. $\sqrt[5]{-x^6y^7}$

30. $\frac{\sqrt{15}}{\sqrt{3}}$

31. $\frac{\sqrt{2a}}{\sqrt{3a}}$, $a \neq 0$

32. $\sqrt{5x} \cdot \sqrt{20x}$, $x \geq 0$

33. $\frac{1}{\sqrt{10}}$

34. $\sqrt[3]{\frac{3}{25}}$

35. $\frac{1-\sqrt{2}}{2+\sqrt{3}}$

36. $\frac{\sqrt{3}+2\sqrt{2}}{3\sqrt{2}+2\sqrt{3}}$

37. $(\sqrt[3]{5})^4$

38. $\sqrt[4]{\sqrt{a}}$, $a \geq 0$

39. $\sqrt[3]{x} \cdot \sqrt{x}$, $x \geq 0$

40. $\sqrt{3} \cdot \sqrt[4]{27}$

41. $(4^{1/2} + 2^{-1})^{-2}$

42. $(a^{-2} + b^{-2})^{-1}$, $a \neq 0$, $b \neq 0$

43. $(a^{-2} + b^{-2})a^{-2}b^{-2}$,
 $a \neq 0$, $b \neq 0$

44. $(x^2 - y^2)^{\frac{1}{2}}$

45. $(a^2 + b^2)^{\frac{1}{2}}$

46. $\sqrt{32} + \sqrt{18} - \sqrt{8} - \sqrt{3}$

47. $\sqrt{63} + \sqrt{24} - 2\sqrt{6} - 2\sqrt{7}$

48. $3\sqrt{50} - 2\sqrt{18}$

49. $2\sqrt{45} + 3\sqrt{20}$

Write in scientific notation

50. 5231554.156

51. 0.000031501

Write in decimal notation

52. $3.22215 \cdot 10^1$

53. $1.20501 \cdot 10^{-2}$

Solutions

- | | | |
|-----------------------|---|-------------------------------------|
| 1. 1 | 21. $x^{\frac{11}{6}}$ | 37. $5^{\frac{4}{3}} = 5^3\sqrt{5}$ |
| 2. 1 | $\frac{2b^2}{3a}$ | 38. $a^{\frac{1}{8}} = \sqrt[8]{a}$ |
| 3. $\frac{1}{81}$ | 22. $\frac{2b^2}{3a}$ | 39. $x^{\frac{5}{6}}$ |
| 4. -16 | 23. 5 | 40. $3^{\frac{5}{4}} = 3^4\sqrt{3}$ |
| 5. -50 | $\frac{cd}{c+d}$ | 41. $\frac{4}{25}$ |
| 6. $10^6 = 1,000,000$ | 24. $\frac{cd}{c+d}$ | $\frac{a^2b^2}{a^2+b^2}$ |
| 7. $2^5 = 32$ | 25. a^2y^3 | 42. $\frac{a^2+b^2}{a^4b^4}$ |
| 8. a^9 | $\frac{1}{16y^{\frac{3}{2}}}$ | 43. $\frac{a^2+b^2}{a^4b^4}$ |
| 9. $8a^3$ | 26. $\frac{1}{16y^{\frac{3}{2}}}$ | 44. does not simplify |
| $\frac{x^4}{y^6}$ | 27. $\frac{4y^4}{a^2}$ | 45. does not simplify |
| 10. $\frac{x^4}{y^6}$ | 28. $3xy^2\sqrt{2x}$ | 46. $5\sqrt{2} - \sqrt{3}$ |
| $\frac{2}{9}$ | 29. $-xy^3\sqrt{xy^2}$ | 47. $\sqrt{7}$ |
| 11. $\frac{2}{9}$ | 30. $\sqrt{5}$ | 48. $9\sqrt{2}$ |
| 12. a | $\frac{\sqrt{6}}{3}$ | 49. $12\sqrt{5}$ |
| 13. $8x^2w^3$ | 31. $\frac{\sqrt{6}}{3}$ | 50. 5.231554156×10^6 |
| 14. y | 32. 10x | 51. 3.1501×10^{-5} |
| 15. -0.2 | $\frac{\sqrt{10}}{10}$ | 52. 32.2215 |
| $\frac{1}{25}$ | 33. $\frac{\sqrt{10}}{10}$ | 53. 0.0120501 |
| 16. $\frac{1}{25}$ | $\frac{\sqrt[3]{15}}{5}$ | |
| 17. 64 | 34. $\frac{\sqrt[3]{15}}{5}$ | |
| 18. 2 | 35. $2 - 2\sqrt{2} - \sqrt{3} + \sqrt{6}$ | |
| $\frac{1}{3}$ | $\frac{-\sqrt{6}+6}{6}$ | |
| 19. $\frac{1}{3}$ | 36. $\frac{-\sqrt{6}+6}{6}$ | |
| 20. 5 | | |