

Completing the Square

Completing the square is another method of solving quadratic equations. It allows trinomials to be factored into two identical factors.

Example: $x^2 + 4x + 4 = (x+2)(x+2)$ or $(x+2)^2$

To complete the square, we need to find the constant term, or last number that will allow us to factor the trinomial into two identical factors. To find the constant term needed, simply take the coefficient of “x”, or the middle number, divide by 2, and then square the quotient. If you have an equation, rather than an expression, the resulting number should be added to **both** sides of the equation.

Example: What would be the constant term used to factor the expression x^2-8x into two identical factors?

Step 1. Take the coefficient of “x”, which is -8, and divide it by two.

$$-8/2=-4$$

Step 2. Take that number and square it.

$$(-4)^2=16$$

Step 3. Adding 16 is the constant term that would allow the expression to be factored into identical factors.

$$x^2-8x+16=(x-4)^2$$

To solve an equation by completing the square requires a couple of extra steps.

Example: Solve by completing the square $x^2+8x+7=0$

Step 1. Move the constant term to the other side of the equation by subtracting from both sides.

$$x^2+8x+7-7=0-7$$

$$x^2+8x=-7$$

Step 2. Complete the square.

$$\left(\frac{8}{2}\right)^2 = (4)^2=16$$

Step 3. Since we are adding 16 to the left side of the equation we **MUST** also add it to the right side.

$$x^2+8x+16=-7+16$$

$$x^2+8x+16=9$$

Step 4. Factor the left side of the equation.

$$x^2+8x+16=9$$

$$(x+4)^2=9$$

HINT: the number inside the factor should always be the same as the number you get when you divide the coefficient of “x” by two! $8/2=4$ and the factor was $(x+4)^2$

Step 5. Take the square root of both sides and solve for x.

$$\sqrt[2]{(x+4)^2} = \sqrt[2]{9}$$

$$x+4=\pm 3$$

$$x=-7 \text{ and } -1$$

To complete the square the coefficient of x^2 must be one. If it is any other number we must first divide the **entire equation** by that number.

Example: Solve by completing the square $4x^2-12x-4=12$

Step 1. Divide the equation by 4 in order to get the leading coefficient to be 1.

$$\frac{(4x^2 - 12x - 4)}{4} = \frac{12}{4}$$

$$x^2-3x-1=3$$

Step 2. Move constants to other side.

$$x^2-3x-1+1=3+1$$

$$x^2-3x=4.$$

Step 3. Complete the square. Take the middle term, -3; divide by 2, and then square.

$$\left(\frac{-3}{2}\right)^2 = \frac{9}{4}$$

Step 4. Add that number to both sides of the equation.

$$x^2 - 3x + \frac{9}{4} = 4 + \frac{9}{4}$$

We need to find a common denominator to add $4 + \frac{9}{4}$, the common denominator would be four so change 4 to $\frac{16}{4}$.

$$\frac{16}{4} + \frac{9}{4} = \frac{25}{4}$$

So, $x^2 - 3x + \frac{9}{4} = \frac{25}{4}$

Step 5. Factor left side. Remember that the number that goes inside the factor is the same as when you divide by two.

$$x^2 - 3x + \frac{9}{4} = \frac{25}{4}$$

$$\left(x + \frac{3}{2}\right)^2 = \frac{25}{4}$$

Step 6. Take the square root of both sides; solve for x.

$$\sqrt{\left(x + \frac{3}{2}\right)^2} = \sqrt{\frac{25}{4}}$$

$$x + \frac{3}{2} = \pm \frac{5}{2}$$

$$x=1 \quad \text{and} \quad x=-4$$

Practice Problems

1. $x^2 + 6x + 5 = 0$

2. $x^2 + 8x - 9 = 0$

3. $x^2 - 6x + 9 = 0$

4. $x^2 + 4x - 7 = 0$

5. $x^2 - 5x - 24 = 0$

6. $x^2 - 8x + 15 = 0$

7. $4x^2 - 4x + 17 = 0$

8. $9x^2 - 12x + 13 = 0$

9. $4x^2 - 4x + 5 = 0$

10. $4x^2 - 8x + 1 = 0$

Answers to Sample Problems

-5 and -1

1 and -9

3 only

$-2 - \sqrt{11}$ and $-2 + \sqrt{11}$

-3 and 8

3 and 5

$\frac{1}{2} - 2i$ and $\frac{1}{2} + 2i$

$\frac{2}{3} + i$ and $\frac{2}{3} - i$

$\frac{1}{2} + i$ and $\frac{1}{2} - i$

$1 + \frac{\sqrt{3}}{2}$ and $1 - \frac{\sqrt{3}}{2}$