

Introduction to Fractions

Fractions represent parts of a whole. The top part of a fraction is called the numerator, while the bottom part of a fraction is called the denominator. The denominator states how many total parts the whole has been divided into. The numerator states how many of those parts are being represented by the fraction.

Example 1: $\frac{1}{8}$ Numerator
Denominator

If I buy a pizza that is cut into 8 pieces and I eat 1 of those pieces, then I can say I ate $\frac{1}{8}$ of the pizza. The numerator represents how many pieces I ate and the denominator represents how many total pieces there were to begin with.

Every number can be written as a fraction. To write a whole number as a fraction, just put a 1 under it.

Example 2: $\frac{5}{1} = 5$

Improper Fractions

A fraction is an improper fraction if the numerator is larger than or equal to the denominator. If a fraction is an improper fraction, this means that the fraction's value is equal to or greater than 1.

Example 3: $\frac{3}{2}$ is an improper fraction.

Mixed Numbers

A mixed number is a whole number with a fraction following it. It is used to represent a fraction whose value is greater than 1.

Example 4: $1\frac{1}{2}$ is a mixed number.

Converting Mixed Numbers into Improper Fractions

- 1) Multiply the whole number by the denominator. Then add that result to the numerator. The answer is the new numerator of the improper fraction.
- 2) The denominator of the improper fraction is the same as the mixed number's denominator.

Example 5: Convert $5\frac{2}{3}$ into an improper fraction.

$$5 \times 3 = 15$$

$$15 + 2 = 17$$

$$\text{Answer: } \frac{17}{3}$$

Converting a Fraction into a Decimal

- 1) Divide the numerator of the fraction by the denominator.
- 2) If you get a remainder, write a zero next to the remainder and continue dividing. Remember to include the decimal place both under the division sign in the numerator and above the division sign in your answer.
- 3) Continue dividing until you get a repeating decimal or until the division ends.

Example 6: Convert $\frac{2}{9}$ into a decimal.

$$\begin{array}{r} 0.222\dots \\ 9 \overline{) 2.000} \\ \underline{-18} \\ 20 \\ \underline{-18} \\ 20 \\ \underline{-18} \\ \vdots \end{array}$$

$$\text{Therefore: } \frac{2}{9} = 0.\overline{2}$$

Example 7: Convert $\frac{5}{8}$ into a decimal.

$$\begin{array}{r} 0.625 \\ 8 \overline{) 5.000} \\ \underline{-48} \\ 20 \\ \underline{-16} \\ 40 \\ \underline{-40} \end{array}$$

$$\text{Therefore: } \frac{5}{8} = 0.625$$

Converting a Mixed Number into a Decimal

- 1) Divide the fraction just as you would for a normal fraction (divide the numerator by the denominator).
- 2) Add this answer to the whole number.

Example 8: Convert $12\frac{5}{8}$ into a decimal.

We have already divided 5 by 8 in Example 7, so we just add 0.625 to 12 which equals 12.625.

$$12\frac{5}{8} = 12.625$$

Converting a Decimal into a Fraction

- 1) Put the entire number into the numerator of the fraction with the decimal point removed.
- 2) Then count the number of digits after the decimal point and place a 1 followed by that many zeros in the denominator in the fraction.
- 3) Reduce the fraction if possible (see the “Simplifying Fractions” section on the next page for additional help in reducing fractions).

Example 9: Convert 3.625 into a fraction

$$3.625 = \frac{3625}{1000}$$

Example 10: Convert 0.31 into a fraction

$$0.31 = \frac{31}{100}$$

Converting Improper Fractions into Mixed Numbers

- 1) Divide the numerator by the denominator.
- 2) The remainder of the division becomes the numerator of the answer.
- 3) The denominator of the improper fraction will be the denominator of the mixed number.

Example 11: Convert $\frac{9}{5}$ into a mixed number.

$$\begin{array}{r} 1R4 \\ 5 \overline{)9} \end{array}$$

$$\underline{-5} \quad \text{Answer: } 1\frac{4}{5}$$

4 (The remainder becomes the numerator of the fraction.)

Simplifying Fractions

To simplify a fraction you try to use smaller numbers in the numerator and denominator to represent the same value as the original fraction. The reason this can be done is that many different fractions have the same value. For example, $\frac{1}{2}$, $\frac{2}{4}$, $\frac{4}{8}$, $\frac{23}{46}$, and $\frac{733}{1466}$ all represent the same value of 0.5. Fractions are often simplified so that you can see if two fractions are equivalent (represent the same value).

To simplify a fraction,

- 1) Look at the numerator and denominator, and see if there is a common factor of the two numbers (GCF).
- 2) Divide both the numerator and denominator by that number.
- 3) See if you can do this again with the resulting fraction.

Example 12: Simplify $\frac{8}{36}$

$$\frac{8}{36} = \frac{4}{18} = \frac{2}{9}$$

First, 8 and 36 have a common factor of 2; when you divide the numerator and the denominator by 2 the fraction becomes $\frac{4}{18}$. Secondly, 4 and 18 have a common factor of 2; when you divide both the numerator and the denominator by 2 the fraction becomes $\frac{2}{9}$. Finally, 2 and 9 do not have any common factors, so the fraction has been simplified.

Example 13: Simplify $\frac{24}{18}$

$$\frac{24}{18} = \frac{12}{9} = \frac{4}{3}$$

First, 24 and 18 have a common factor of 2; when you divide the numerator and the denominator by 2 the fraction becomes $\frac{12}{9}$. Secondly, 12 and 9 have a common factor of 3; when you divide the numerator and the denominator by 3 the fraction becomes $\frac{4}{3}$. Finally, 4 and 3 do not have any common factors, so the fraction has been simplified.

Example 14: Simplify $\frac{11}{23}$

Since 11 and 23 do not have any common factors, $\frac{11}{23}$ is simplified.

Multiplying Fractions

Multiplying is one of the easiest things you can do with fractions. Remember, you must change mixed numbers into improper fractions before you can multiply them!

To do this,

- 1) Multiply the two numerators together. The result of the multiplication is the numerator of the answer.
- 2) Multiply the two denominators together. The result of the multiplication is the denominator of the answer.
- 3) Always simplify your answer if possible.

Example 15: $\frac{3}{4} \times \frac{2}{3}$

$$3 \times 2 = 6 \quad \text{Multiply the numerators.}$$

$$4 \times 3 = 12 \quad \text{Multiply the denominators.}$$

$$\text{Answer: } \frac{6}{12} \text{ or } \frac{1}{2} \quad \text{Simplify the fraction.}$$

Example 16: $2\frac{1}{3} \times \frac{-2}{5}$

$$\frac{7}{3} \times \frac{-2}{5}$$

$$7 \times -2 = -14 \quad \text{Multiply the numerators.}$$

$$3 \times 5 = 15 \quad \text{Multiply the denominators.}$$

$$\text{Answer: } \frac{-14}{15}$$

Convert to improper fractions.

Dividing Fractions

Remember, you must change mixed numbers into improper fractions before you can divide them! To divide fractions, you must change the division into multiplication.

To do this,

- 1) The 1st fraction remains the same.
- 2) Change the (\div) sign into a (\times) sign.
- 3) Use the reciprocal of the 2nd fraction.
- 4) Multiply the fractions.

Example 17: $\frac{3}{4} \div \frac{2}{5}$

$$\frac{3}{4} \times \frac{5}{2}$$

$$\text{Answer: } \frac{15}{8}$$

Copy the first fraction, change to multiplication, and flip the second fraction.
Multiply the fractions.

Example 18: $-8 \div \frac{2}{5}$

$$\frac{-8}{1} \div \frac{2}{5}$$

$$\frac{-8}{1} \times \frac{5}{2}$$

$$\text{Answer: } \frac{-40}{2} \text{ or } \frac{-20}{1} \text{ or } -20$$

Write 8 as a fraction.
Copy the first fraction, change to multiplication, and flip the second fraction.
Multiply the fractions.

Example 19: $-4\frac{1}{9} \div 5\frac{3}{4}$

$$\frac{-37}{9} \div \frac{23}{4}$$

$$\frac{-37}{9} \times \frac{4}{23}$$

Answer: $\frac{-148}{207}$

Convert to improper fractions.

Copy the first fraction, change to multiplication, and flip the second fraction.

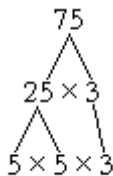
Multiply the fractions.

Least Common Multiple (LCM)

The least common multiple of two numbers is the smallest number that is a multiple of both numbers. To find the LCM of two numbers,

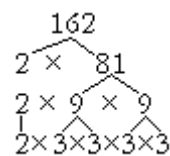
- Factor each number. An easy way to factor a number is by creating a factor tree. Factor trees work by breaking the number you are factoring into two numbers that when multiplied together, equal the number you are factoring. When you cannot break the numbers down any more, you only have prime numbers, the number is completely factored.

Example 20: Factor 75



Answer: $5 \times 5 \times 3$

Example 21: Factor 162



Answer: $2 \times 3 \times 3 \times 3 \times 3$

- Once you have factored both numbers, then you want to see what factors are shared between the two numbers. List the shared factors.
- List the factors that are not shared by the numbers.
- Multiply the factors that you have listed to get the LCM.

Example 22: Find the LCM of 7 and 14



Since the only thing that can multiply to get 7 is 1 and 7, 7 is already a prime number (and thus cannot be factored).

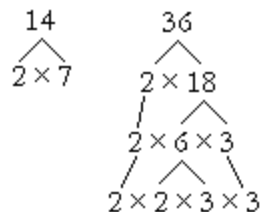
7 and 14 share a 7, so list one 7.

The only number not shared is a 2 from the 14, so list one 2.

Multiply the numbers you listed.

$7 \times 2 = 14$, so the LCM of 7 and 14 is **14**.

Example 23: Find the LCM of 14 and 36.



14 and 36 share one 2, so list one 2.

One 7 is not shared from the 14, so list one 7.

One 2 and two 3's are not shared from the 36, so list one 2 and two 3's.

Multiply the numbers you listed.

$2 \times 7 \times 2 \times 3 \times 3 = 252$ so the LCM of 14 and 36 is **252**.

Comparing Fractions

To compare fractions,

- 1) You must have the same denominators in both fractions. To do this,
 - a) Find the LCM between the current denominators of the two fractions you wish to compare. The LCM will be the new denominator of the fractions.
 - b) The numerators must be changed as well so that the value of the fraction is not changed. Find out what number multiplied by the old denominator of the fraction equals the LCM. (Divide the LCM by the current denominator to find what to multiply by.) Now multiply that number by the numerator to get the new numerator.
- 2) Compare the numerators of the two fractions to see which is larger.

Example 24: Which fraction is larger: $\frac{7}{27}$ or $\frac{5}{21}$?

The LCM of 27 and 21 is 189.

$$\text{So } \frac{7}{27} = \frac{?}{189} \quad \text{So } \frac{5}{21} = \frac{?}{189}$$

$$27 \times 7 = 189 \quad 21 \times 9 = 189$$

$$\text{So } \frac{7}{27} \times \frac{7}{7} = \frac{49}{189} \quad \text{So } \frac{5}{21} \times \frac{9}{9} = \frac{45}{189}$$

$49 > 45$ so $\frac{7}{27}$ is the larger fraction.

Adding and Subtracting Fractions

To add or subtract two fractions,

- 1) The denominators of both fractions must be the same. If they are not, you must change them so that they are equal. To do this,
 - a. Find the LCM of the denominators of the fractions you are adding or subtracting. The LCM will be the new denominator of the fractions.
 - b. The numerators must be changed as well so that the value of the fraction is not changed. Find out what number multiplied by the old denominator of the fraction equals the LCM (Divide the LCM by the current denominator to find out what to multiply by). Now multiply that number by the numerator to get the new numerator.
 - c. Add the numerators of the fractions, keeping the same denominator.
 - d. Simplify, if possible.

Example 25: $\frac{11}{12} + \frac{7}{8}$

The LCM of 12 and 8 is 24. So this is what the denominators of the fractions will be changed to.

$$\text{So } \frac{11}{12} = \frac{?}{24} \quad \text{So } \frac{7}{8} = \frac{?}{24}$$

$$12 \times 2 = 24 \quad 8 \times 3 = 24$$

$$\text{So } \frac{11}{12} \times \frac{2}{2} = \frac{22}{24} \quad \text{So } \frac{7}{8} \times \frac{3}{3} = \frac{21}{24}$$

Now, add the numerators.

$$\frac{22}{24} + \frac{21}{24} = \frac{43}{24}$$

Answer: $\frac{43}{24}$

Summary

A *fraction* represents a part of a whole. A *mixed number* is a whole number and a fraction written together. An *improper fraction* is a mixed number written as a fraction. It is found by multiplying the whole number by the denominator and adding that answer to the numerator. (Leave the denominator the same).

To *simplify a fraction*, factor the numerator and denominator, and then divide the original numbers by the GCF (greatest common factor) of the two numbers.

When you are *multiplying or dividing fractions*, it does not matter if the denominators match. Simply multiply the numerators together and the denominators together.

When you are *adding or subtracting fractions*, remember that you must have the common (same) denominators before you add or subtract the numerators.

If you found this handout helpful, you may also like to look at these handouts:

- Decimals, Fractions, and Percents
- Into. to Word Problems

Practice Problems

Convert the following fractions into Mixed Numbers.

- | | | | |
|-------------------|--------------------|----------------------|---------------------|
| 1. $\frac{20}{3}$ | 4. $\frac{10}{9}$ | 7. $\frac{-126}{10}$ | 10. $\frac{43}{-8}$ |
| 2. $\frac{17}{4}$ | 5. $\frac{31}{7}$ | 8. $-\frac{89}{11}$ | 11. $\frac{1}{-8}$ |
| 3. $\frac{5}{2}$ | 6. $\frac{5}{-16}$ | 9. $\frac{-51}{17}$ | 12. $\frac{-35}{7}$ |

Convert the following Mixed Numbers fractions into Fractions.

- | | | | |
|--------------------|-----------------------|-----------------------|---------------------|
| 13. $1\frac{2}{3}$ | 16. $3\frac{2}{9}$ | 19. $-8\frac{3}{11}$ | 22. $8\frac{5}{8}$ |
| 14. $4\frac{1}{4}$ | 17. $-8\frac{1}{5}$ | 20. $-4\frac{12}{17}$ | 23. $1\frac{7}{20}$ |
| 15. $2\frac{1}{2}$ | 18. $-12\frac{6}{10}$ | 21. $-7\frac{5}{8}$ | 24. 8 |

Simplify the following fractions completely.

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|---------------------|----------------------|-----------------------|------------------------|
| 25. $\frac{84}{4}$ | 28. $\frac{11}{121}$ | 31. $\frac{-3}{14}$ | 34. $\frac{-34}{80}$ |
| 26. $\frac{20}{42}$ | 29. $\frac{6}{36}$ | 32. $\frac{-18}{26}$ | 35. $\frac{-128}{256}$ |
| 27. $\frac{20}{30}$ | 30. $\frac{8}{-24}$ | 33. $-\frac{16}{128}$ | 36. $\frac{180}{400}$ |

Multiply or Divide. Write answers as improper fractions.

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|--------------------------------------|--------------------------------------|---|---|
| 37. $\frac{2}{3} \times \frac{1}{4}$ | 40. $\frac{7}{13} \div \frac{4}{3}$ | 43. $\frac{2}{9} \times -1\frac{1}{2}$ | 46. $3 \times -1\frac{1}{11}$ |
| 38. $\frac{5}{2} \div \frac{1}{7}$ | 41. $\frac{4}{7} \times 12$ | 44. $-7\frac{1}{3} \div -4\frac{1}{4}$ | 47. $\frac{-7}{5} \times \frac{10}{90}$ |
| 39. $\frac{4}{7} \times \frac{2}{5}$ | 42. $1\frac{7}{11} \div \frac{8}{3}$ | 45. $\frac{14}{3} \times \frac{5}{-21}$ | 48. $4\frac{14}{3} \div \frac{25}{-16}$ |

Find the Least Common Multiple of the following numbers

- | | | | |
|------------|------------|-------------|------------|
| 49. 8, 16 | 52. 25, 35 | 55. 24, 72 | 58. 34, 51 |
| 50. 16, 24 | 53. 20, 34 | 56. 24, 120 | 59. 15, 70 |
| 51. 11, 21 | 54. 2, 9 | 57. 64, 256 | 60. 15, 20 |

Which fraction is larger?

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|----------------------------------|-----------------------------------|-------------------------------------|---|
| 61. $\frac{2}{3}, \frac{4}{5}$ | 64. $\frac{7}{12}, \frac{10}{16}$ | 67. $\frac{18}{-24}, \frac{-5}{6}$ | 70. $\frac{-19}{26}, \frac{-9}{13}$ |
| 62. $\frac{22}{4}, \frac{23}{5}$ | 65. $\frac{6}{7}, \frac{42}{49}$ | 68. $\frac{-6}{9}, \frac{120}{180}$ | 71. $\frac{-1}{1000}, \frac{-5}{2000}$ |
| 63. $\frac{-3}{8}, \frac{1}{8}$ | 66. $1\frac{2}{3}, \frac{20}{12}$ | 69. $\frac{-1}{20}, \frac{-3}{40}$ | 72. $\frac{130}{2259}, \frac{-154}{3000}$ |

Add or Subtract. Write answers as improper fractions.

- | | | | |
|-----------------------------------|-----------------------------------|-------------------------------------|---------------------------------------|
| 73. $\frac{2}{3} + \frac{1}{7}$ | 76. $\frac{4}{6} + \frac{-7}{4}$ | 79. $2\frac{8}{7} + 7\frac{2}{3}$ | 82. $-2\frac{9}{11} + 8$ |
| 74. $\frac{3}{11} - \frac{8}{2}$ | 77. $\frac{9}{15} + \frac{5}{16}$ | 80. $7\frac{1}{4} - 3\frac{3}{5}$ | 83. $\frac{9}{11} + 2\frac{7}{22}$ |
| 75. $\frac{7}{10} + \frac{8}{15}$ | 78. $\frac{7}{-8} + \frac{8}{10}$ | 81. $-8\frac{2}{21} - 4\frac{1}{7}$ | 84. $1\frac{59}{11} + 8\frac{15}{12}$ |

Add or Subtract. Write answers as mixed numbers.

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|--------------------------------------|-------------------------------------|-----------------------------------|------------------------------------|
| 85. $\frac{2}{3} - \frac{1}{7}$ | 88. $\frac{-2}{6} - \frac{9}{10}$ | 91. $\frac{1}{6} + \frac{5}{8}$ | 94. $\frac{1}{-4} + \frac{3}{12}$ |
| 86. $\frac{621}{10} + \frac{57}{18}$ | 89. $-\frac{9}{15} + \frac{20}{7}$ | 92. $\frac{9}{20} - \frac{5}{12}$ | 95. $2\frac{1}{4} - 8\frac{3}{12}$ |
| 87. $\frac{5}{12} - 8\frac{7}{16}$ | 90. $-\frac{1}{10} + -\frac{1}{12}$ | 93. $\frac{7}{25} + \frac{3}{25}$ | 96. $5\frac{1}{9} + 4\frac{7}{12}$ |

Answers

Convert the following into Mixed Numbers.

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|-------------------|--------------------|--|---------------------|
| 1. $6\frac{2}{3}$ | 4. $1\frac{1}{9}$ | 7. $-12\frac{6}{10}$ or $-12\frac{3}{5}$ | 10. $-5\frac{3}{8}$ |
| 2. $4\frac{1}{4}$ | 5. $4\frac{3}{7}$ | 8. $-8\frac{1}{11}$ | 11. $-\frac{1}{8}$ |
| 3. $2\frac{1}{2}$ | 6. $-\frac{5}{16}$ | 9. -3 | 12. -5 |

Convert the following Mixed Numbers into Fractions.

- | | | | |
|--------------------|--|----------------------|---------------------|
| 13. $\frac{5}{3}$ | 16. $\frac{29}{9}$ | 19. $-\frac{91}{11}$ | 22. $\frac{69}{8}$ |
| 14. $\frac{17}{4}$ | 17. $-\frac{41}{5}$ | 20. $-\frac{80}{17}$ | 23. $\frac{27}{20}$ |
| 15. $\frac{5}{2}$ | 18. $-\frac{126}{10}$ or $-\frac{63}{5}$ | 21. $-\frac{61}{8}$ | 24. 8 |

Simplify the following fractions completely.

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|---------------------|--------------------|---------------------|----------------------|
| 25. 21 | 28. $\frac{1}{11}$ | 31. $-\frac{3}{14}$ | 34. $-\frac{17}{40}$ |
| 26. $\frac{10}{21}$ | 29. $\frac{1}{6}$ | 32. $-\frac{9}{13}$ | 35. $-\frac{1}{2}$ |
| 27. $\frac{2}{3}$ | 30. $-\frac{1}{3}$ | 33. $-\frac{1}{8}$ | 36. $\frac{9}{20}$ |

Multiply or Divide. Write answers as improper fractions.

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|--------------------|---------------------|---------------------|-----------------------|
| 37. $\frac{1}{6}$ | 40. $\frac{21}{52}$ | 43. $-\frac{1}{3}$ | 46. $-\frac{36}{11}$ |
| 38. $\frac{35}{2}$ | 41. $\frac{48}{7}$ | 44. $\frac{88}{51}$ | 47. $-\frac{7}{45}$ |
| 39. $\frac{8}{35}$ | 42. $\frac{27}{44}$ | 45. $-\frac{10}{9}$ | 48. $-\frac{416}{75}$ |

Find the Least Common Multiple of the following numbers.

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|---------|---------|---------|---------|
| 49. 16 | 52. 175 | 55. 72 | 58. 102 |
| 50. 48 | 53. 340 | 56. 120 | 59. 210 |
| 51. 231 | 54. 18 | 57. 256 | 60. 60 |

Which fraction is larger?

- | | | | |
|--------------------|---------------------|-----------------------|------------------------|
| 61. $\frac{4}{5}$ | 64. $\frac{10}{16}$ | 67. $-\frac{18}{24}$ | 70. $-\frac{9}{13}$ |
| 62. $\frac{22}{4}$ | 65. Equal | 68. $\frac{120}{180}$ | 71. $-\frac{1}{1000}$ |
| 63. $\frac{1}{8}$ | 66. Equal | 69. $-\frac{1}{20}$ | 72. $\frac{130}{2259}$ |

Add or Subtract. Write answers as improper fractions.

- | | | | |
|----------------------|----------------------|-----------------------|----------------------|
| 73. $\frac{17}{21}$ | 76. $-\frac{13}{12}$ | 79. $\frac{227}{21}$ | 82. $\frac{57}{11}$ |
| 74. $-\frac{41}{11}$ | 77. $\frac{73}{80}$ | 80. $\frac{73}{20}$ | 83. $\frac{69}{22}$ |
| 75. $\frac{37}{30}$ | 78. $-\frac{3}{40}$ | 81. $-\frac{257}{21}$ | 84. $\frac{687}{44}$ |

Add or Subtract. Write answers as mixed numbers.

- | | | | |
|----------------------|----------------------|---------------------|----------------------|
| 85. $\frac{11}{21}$ | 88. $-1\frac{7}{30}$ | 91. $\frac{19}{24}$ | 94. 0 |
| 86. $65\frac{4}{15}$ | 89. $2\frac{9}{35}$ | 92. $\frac{1}{30}$ | 95. -6 |
| 87. $-8\frac{1}{48}$ | 90. $\frac{-11}{60}$ | 93. $\frac{2}{5}$ | 96. $9\frac{25}{36}$ |