Logarithms

Natural exponential function

$f(x) = e^x$

e is called **Euler's number**. Like π, Euler's number is an irrational number; it's decimal equivalent is a never-ending sequence of digits:

e ≈ 2.71828182845904523…

Logarithmic functions answer the following question "to what power must I raise a number to in order to get a specific number?"

example: $\log_e x = n$ ≡ $e^n = x$

$\log_3 81 = x$ ≡ $3^x = 81$; x = 4

$\log_2 x = -5$ ≡ $2^{-5} = x$; x = $\frac{1}{32}$

$\log_x 125 = 3$ ≡ $x^3 = 125$; x = 5

$\log_a 1 = x$ ≡ $a^x = 1$; x = 0 (remember $x^0 = 1$)

Base 10 or Common Log (no base is shown; log)

log 1000 = 3 ≡ $10^3 = 1000$

Base e or Natural Logarithm

ln x or $\log_e x$

Inverse Relationship

$\log_a(a^x) = a^{\log_z x} = x$

$\log(10^x) = 10^{\log x} = x$

$\ln(e^x) = e^{\ln x} = x$

Properties of Logarithms

1) log (xy) = log x + log y

ex: ln 15 = ln (3*5) = ln 3 + ln 5

2) $\log x^a = a \log x$

ex: $\ln x^4$ = ln(x·x·x·x) = ln x + ln x + ln x + ln x = 4ln x

3) $\log \frac{x}{y} = \log x - \log y$

ex: $\log \frac{x}{y} = \log x + \log y^{-1} = \log x + (-1)\log y = \log x - \log y$

Example Logarithm Problems

1. Simplify:

   $\log(2x^2y^3)$

   apply log property #1

   $\log(2)+\log(x^2)+\log(y^3)$

   apply log property #3

   $\log(2)+2\log(x)+3\log(y)$

2. Simplify:

   $\log_2(\frac{\sqrt[3]{x+1}}{2x})$

   apply log property #3

   $\log_2(\sqrt[3]{x+1})-\log_2(2x)$

   apply log properties #2 and #1

   $\frac{1}{3}\log_2(x+1)-(\log_2(2)+\log_2(x))$

   remember that $\log_x x=1$

   $\frac{1}{3}\log_2(x+1)-\log_2(x)-1$

3. Write in terms of one logarithmic function:

   $3\ln(x)-2\ln(x+1)$

   apply log property #2

   $\ln(x^3)-\ln(x+1)^2$

   apply log property #3

   $\ln\left(\frac{x^3}{(x+1)^2}\right)$

4. Write in terms of one logarithmic function:

   $\log(x)-2\log(y)-\log(z)+3\log(w)$

   exponent rule: (+) term in numerator; (-) terms in denominator

   $\log\left(\frac{xw^3}{y^2z}\right)$

Practice Problems

Solve for x:

1. $3^x = 243$
2. $5^x = 13$
3. $2e^{3x} - 3 = 15$
4. $e^{2x} - 2e^x - 15 = 0$
5. $\log_7(x) = 2$
6. $\log(3x) = \frac{1}{4}$
7. $(\ln x)^2 - 2\ln(x^4) = 20$
8. $\log(\frac{1}{100}) = x$
9. $\log_x(5) = 2$
10. $\log_8(x) = \frac{4}{3}$
11. $\ln(e^{17}) = x$
12. $\ln(2x) + \ln(5) = 3$
13. $6^x = 216$

Use a calculator to evaluate:

14. $\log_2 15$

Rewrite in terms of one logarithmic function:

15. $4\ln(x+3y) - 2\ln(z) + \frac{1}{2}\ln(w)$

Answers to Practice Problems

1. $x = 5$
2. $x \approx 1.59369$
3. $x \approx 0.73241$
4. $x = \ln(5)$
5. x = 49
6. $x \approx 0.5928$
7. x = e^{10} or $\frac{1}{e^2}$
8. x = -2
9. $x = \sqrt{5}$
10. x = 16
11. x = 17
12. x = $\frac{e^3}{10}$
13. x = 3
14. 3.907
15. $\ln \frac{(x+3y)^4 \cdot \sqrt{w}}{z^2}$